Problem 3

$$\frac{1}{a} + \frac{a}{b} + \frac{1}{ab} = 1$$

$$\frac{b}{ab} + \frac{a^2}{ab} + \frac{1}{ab} = 1$$

$$a^2 + b + 1 = ab$$

$$a^2 + 1 = b(a-1)$$

$$\frac{a^2+1}{a-1} = b$$

$$\frac{a^2-1}{a-1} + \frac{2}{a-1} = b$$

$$a + 1 + \frac{2}{a-1} = b$$

Therefore, the only integers a for which b is an integer are 2 and 3, where b is 5 in both cases. Therefore, the only possible integer pairs (a,b) that satisfy $\frac{1}{a} + \frac{a}{b} + \frac{1}{ab} = 1$ are $\boxed{(2,5) \text{ and } (3,5).}$

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